Physica A 525 (2019) 433–442

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Binary opinion dynamics on signed networks based on Ising model

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HIGHLIGHTS

- Study the binary opinion dynamics on signed networks by aid of the Ising model.
- The proportion and distribution of negative edges have a fundamental effect on the evolutionary result of public opinion.
- There exists the critical ratio of negative edges in the evolution of opinion.

ARTICLE INFO

Article history:
Received 25 October 2018
Received in revised form 24 January 2019
Available online 19 March 2019

Keywords:
Signed network
Binary opinion dynamics
Ising model
Negative edges
Network structural balance

ABSTRACT

The evolution dynamics of public opinion is a hot issue in complex networks research. And the Ising model is the earliest classic dynamic model of public opinion. Although signed networks can describe amicable and antagonistic relationship in complex real-world systems accurately, and the research on dynamic process of public opinion evolution on signed networks is valuable, few people have paid attention to that. Previous methods for opinion diffusion cannot be applied to signed network directly, which ignore the important information contained in the negative edges. In this paper, the binary opinion dynamics on signed networks has been modeled by aid of the Ising model. The model is applied both to the synthetic and real-world signed networks. We observe that the proportion and distribution of negative edges have a fundamental effect on the evolutionary result of public opinion on signed networks. There exists the critical ratio. When the proportion of negative edges in the network exceeds the critical ratio, there appears to be a completely different evolutionary result, and the distribution of negative edges affects the value of the critical ratio. In addition, we study the network structural balance in the evolutionary process of opinion on signed networks as complementary. Our findings can deepen the understanding of the evolutionary process of binary opinion in real signed social systems.

1. Introduction

Complex network research has attracted remarkable attention [1–3], which kind of method can abstract complex systems in real world into succinct mathematical expressions. It provides a more effective way to express the structure of real system, than most of the previous methods, and helps pave the way analyzing the organization, function and dynamics of the systems properly [4].

In the study of complex networks, the dynamic evolution process of public opinion is a hot issue. The dynamic evolution models of public opinion mainly focus on when and how the opinion spreading process leads to the final formation of

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https://doi.org/10.1016/j.physa.2019.03.011
0378-4371/

public opinion. But which method is suitable for describing this dynamic process? Aiming at this problem, the scholars have made a lot of attempts, who use methods in the fields of statistical physics [5], social dynamics [6] and social psychology [7]. The Ising model is the earliest widely used dynamics model of public opinion to describe the phase transition of binary opinions [8]. There have been some achievements on applying the Ising model to the research of public opinion dynamics on complex networks [9–11].

Signed network, which is an important development of complex network theory, refers to the network whose edges have positive or negative attributes. Negative edge in signed network usually represents hostility, conflict, opposition and distrust between individuals or organizations, while positive edge usually represents some positive attitudes or good relationships inversely. These antagonistic relationships exist in many real complex systems, especially in the fields of information, biology and society [12,13]. Research on signed network structure can effectively help understand the essential characteristics of real systems. So when people study signed networks, the function of negative edges is always an important topic that remains to be worked out [14,15].

Applying the Ising model to signed networks is quite a valuable research that few people pay attention to. There has been little work about that, which mainly focus on the physical properties of the theoretical model, such as the changes of the magnetic moment and temperature of the system. Quoting [16]: "the signed graph depicting the liking relations among a group of people will, then, also depict the potential influence structure of the group". For the signed networks, the opinion evolutionary process is more closely related to the structural features of the network, and the positive and negative relationships both have significant impacts on the opinion diffusion results [17]. As mentioned in a study by Jager and Amblard [18], when people vote, they often vote against certain policies, not because they disagree with the policy, but rather because they dislike the political leaders who are advocating the policy. In other words, we assume that the process of opinion diffusion, especially in social networks, is determined by the relationship between users [19]. And the relationship between users corresponds to the interaction between magnetic needles in the Ising model, so this model can be effectively extended to the research of public opinion dynamics on signed networks.

In this work, we study the influence of the negative edges on the evolution process of the Ising model on signed networks. We model the binary opinion dynamics on signed networks by aid of the Ising model, so as to observe the influence of the structural characteristics of the signed network on the public opinion evolutionary results, as well as the correlation with the network structural balance. Compared with previous works, our main contribution is that we focus on the influence of negative edges and network structure of signed networks on the opinion evolutionary process, instead of the physical properties of the system, such as magnetic moment.

The paper is organized as follows. In Section 2, we extend the Ising model to signed networks and analyze the dynamic process in homogeneous networks with signed relationships. In Section 3, we present the results of a series of simulations in different signed networks, and talk about the network structural balance. Finally, the conclusion and discussion are given in Section 4.

2. The binary opinion dynamic model on signed networks

The Ising model is a simple model which describes a highly idealized ferromagnet. The Ising model assumes that ferromagnetic materials are composed of a regular array of magnetic needle, each of which has only two directions (spins). The adjacent needles interact with each other through energy constraints, and at the same time they undergo random magnetic transformations (up and down transformations) due to the interference of environmental thermal noise.

In this work, the Ising model is extended to signed networks to model the binary opinion dynamics. In our model, nodes of signed network interact with each other and constantly adjust their opinions, just as magnetic needles adjust the spin direction due to their interaction in Ising model. And this leads to the continuous changes of the overall opinion distribution in signed network until the system reaches steady state. This model requires only two possible states for each node in the network, that is, public opinion in the system is limited to two discrete values, representing positive and negative views respectively. The state of node is expressed by S_i, whose value can be +1 or -1. According to the evolutionary mechanism of the Ising model, the system is always in the process of pursuing the state with the lowest value of Hamiltonian energy function.

Actually, there are many other factors which influence opinion spreading. For example, the opinion of an individual in social networks may be affected by real-world relationships or external influences. In this work, for the sake of simplicity, the following Settings are proposed:

(i) We first place the focus on the interaction between nodes in the system, and the influence of external field is temporarily ignored.

(ii) To rigorously ensure that the system can spontaneously evolve into the lowest state of energy, the ambient temperature infinitely approaches absolute zero, which is a given value of 10^{-30}.

Under the guarantee of the above two Settings, the Hamiltonian function of the signed network is defined as

$$H_{\{S_i\}} = -\sum_{(i,j)} J_{ij} S_i S_j = -\sum_{(i,j)} a_{ij} S_i S_j \tag{1}$$

where the S_i and S_j represent the state of node i and node j, respectively. And a_{ij} represents the value in the location (i, j) of the adjacency matrix of the network, which corresponds to the coupling coefficient $J_{i,j}$ in the classic Ising model.

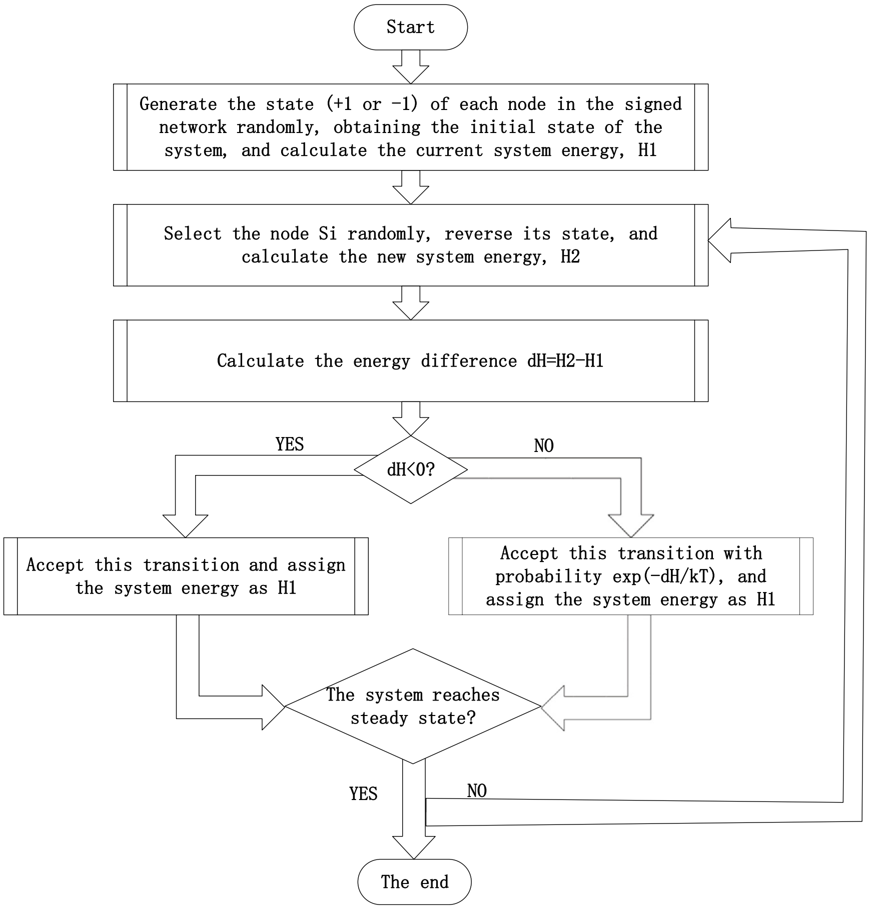

Fig. 1. The Monte Carlo simulation of the binary opinion dynamic model on signed networks based on Ising Model.

And $H_{\{S_i\}}$ represents the total energy under the state combination $\{S_i\}$. Therefore, it can be seen that the energy constraint between nodes in the network is as follows: when two nodes are connected by positive edge, if they are in the same state, the system energy decreases, otherwise the system energy increases. The opposite is true when two nodes are connected by negative edges.

We show the whole dynamic process of the Ising model on signed networks in Fig. 1. At the beginning of evolution, we produce the initial system state according to initial negative node proportion ρ_0 and other parameters, and calculate the current system energy H1. Select a node randomly, reverse its state S_i, and calculate the new system energy H2 and the energy difference δH. Then we need a comparison. If the energy reduces, that is, $\delta H < 0$, the transition will be accepted. If the energy does not change or increases, that is, $\delta H \geq 0$, the transition will be accepted by the probability of $e^{-\delta H/(k_B T)}$. This process will be repeated, which means the state combination of nodes in the network is constantly updated, until the system evolves to a steady state, where the nodes states in the network no longer change and the system reaches the dynamic equilibrium state with the lowest energy. This is the whole process of Monte-Carlo simulation of the Ising model on signed networks. It should be noted that the structure of the edges in the network does not change during the whole evolutionary process.

3. Results

In order to observe the function of negative edges, in this section we apply our model to a series of synthetic signed networks with various known structure, including community structures and random structure. After that, the method is also applied to two real-world signed networks and their corresponding random signed networks.

3.1. Synthetic signed networks

We used synthetic signed networks with various community structures and synthetic random signed networks without distinct community structure for comparative experiments. Therefore, the influence of the proportion and position of the negative edges on the evolutionary results can be observed.

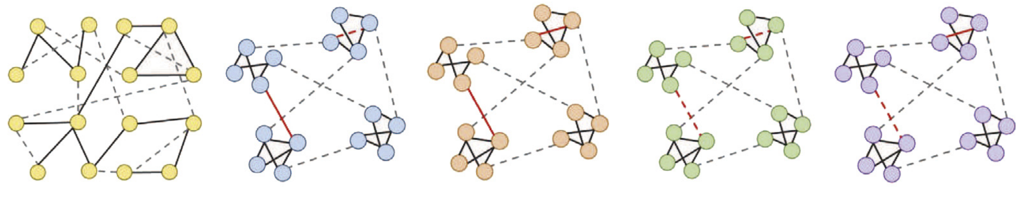

Fig. 2. The five kinds of synthetic networks with 16 nodes (n=16). (a) ERs structure represents the random signed networks without distinct community structure. (b)∼(e) are all generated by one model with controlled community structure, which is controlled by following parameters: $c,\ n,\ \langle k\rangle,\ pin,\ p\_p,\ p\_n$. (b) BMs structure represents the network structure with $p\_p > 0$ and $p\_n > 0$. (c) GGSs structure represents the network structure with $p\_p > 0$ and $p\_n = 0$. (d) SPPs structure represents the network structure with $p\_p = 0$ and $p\_n > 0$. (e) TARs structure represents the network structure with $p\_p = 0$ and $p\_n = 0$.

In this work, we adopt the synthetic signed networks with controlled community structure proposed by Yang et al. [20]. The communities in signed networks are defined as the groups of nodes within which the positive links are dense and between which negative links are dense. In these synthetic signed networks, most positive edges exist within communities and most negative edges exist between the communities. This synthetic signed network generated model is controlled by the following parameters: $c,\ n,\ \langle k\rangle,\ pin,\ p\_p,\ p\_n$, where c is the number of communities in the signed network, n is the total number of nodes in the network (the number of nodes in each community is same), $\langle k\rangle$ is the average degree of each node, pin indicates the probability of each node connecting other nodes in the same community, $1 - pin$ indicates the probability of each node connecting other nodes in the other communities, p_p denotes the proportion of positive links existing between communities, and p_n represents the proportion of negative links existing within communities. So we can calculate the proportion of negative edges as

$$p = pin * p\_n + (1 - pin) * (1 - p\_p) \tag{2}$$

In this network model, once the parameters $c,\ n,\ \langle k\rangle$ are determined, the network structure of signed network only depends on parameters pin, p_p, p_n. Therefore, four networks are generated by adjusting parameters:

(1) $p\_p = 0.1$ and $p\_n = 0.1$, that is, positive and negative edges exist within and between communities. (To illustrate this network structure, we used 16 nodes as an example to make schematic diagram, shown in Fig. 2(b) BMs.)

(2) $p\_p = 0.1$ and $p\_n = 0$, that is, there are positive and negative edges between communities, but only positive edges within communities. (The example of 16 nodes network is shown in Fig. 2(c) GGSs.)

(3) $p\_p = 0$ and $p\_n = 0.1$, that is, there are positive and negative edges within communities, but only negative edges between communities. (The example of 16 nodes network is shown in Fig. 2(d) SPPs.)

(4) $p\_p = 0$ and $p\_n = 0$, that is, there are only positive edges within communities and negative edges between community. (The example of 16 nodes network is shown in Fig. 2(e) TARs.)

As for the comparison model, the synthetic random signed network without distinct community structure, its network structure is illustrated by the example of 16 nodes network shown in Fig. 2(a) ERs. This network's generation mechanism includes the following parameters: $n,\ \langle k\rangle,\ p$, where n is the total number of nodes in the network, $\langle k\rangle$ is the average degree of each node, and p indicates the probability of each node connecting other nodes with negative edge. The network has no community structure, and the positive and negative edges are randomly distributed in the network.

Therefore, we use these generation mechanisms above to generate five kinds of networks with different structural characteristics, and use each kind of networks to carry out public opinion evolution according to the model described in Section 2. In the simulation, we set $c = 4,\ n = 128,\ \langle k\rangle = 16$, and adjusts the ratio of negative edges and initial negative points at intervals of 0.1 to get different sets of parameters. Under each set of parameters for each network structure, we generated 50 networks, and each network evolves for 1000 times, so as to avoid the effect of contingency on experimental results.

We show the simulation results in Figs. 3 and 4. As we can see, Fig. 3 represent the evolutionary results on networks with even numbers of communities while Fig. 4 represent the results about odd numbers, both containing the results of five different network structures. The heat maps show the evolutionary results under different parameter sets, plotted by initial negative node ratio on the horizontal axis and negative edge ratio on the vertical, and the color represents the steady-state negative node ratio. The bubble maps show the critical value of negative edge ratio under different initial negative node ratios, where the size of a square bubble represents the standard deviation of its corresponding data set.

According to the results shown in Figs. 3 and 4, the following conclusions are obtained:

(1) *(Critical Ratio)* There is a critical negative edge ratio, σ. The critical ratio stands for a turning point, which means, when the proportion of negative edges in the network exceeds the critical ratio, there appear to be two distinct evolutionary results. The detection of the critical ratio depends on a change-points detection method, pettitt, which is a nonparametric test method [21].

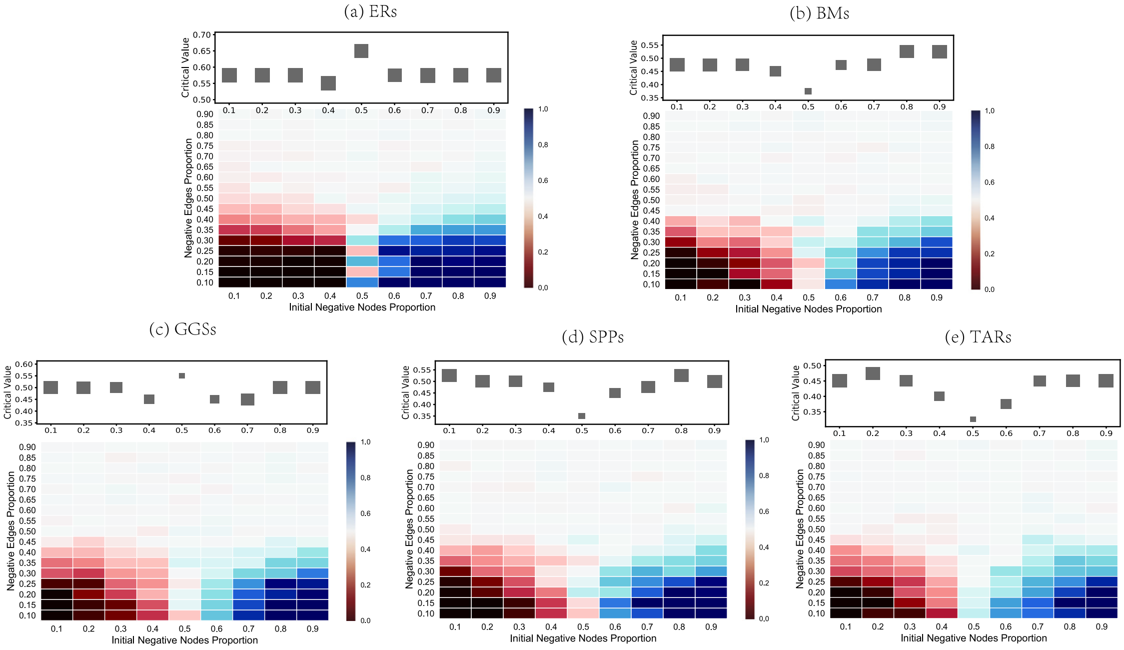

Fig. 3. The evolutionary results on networks with even numbers of communities. (a)∼(e) represent the results on ER structure, BM structure, GGS structure, SPP structure and TAR structure networks, respectively.

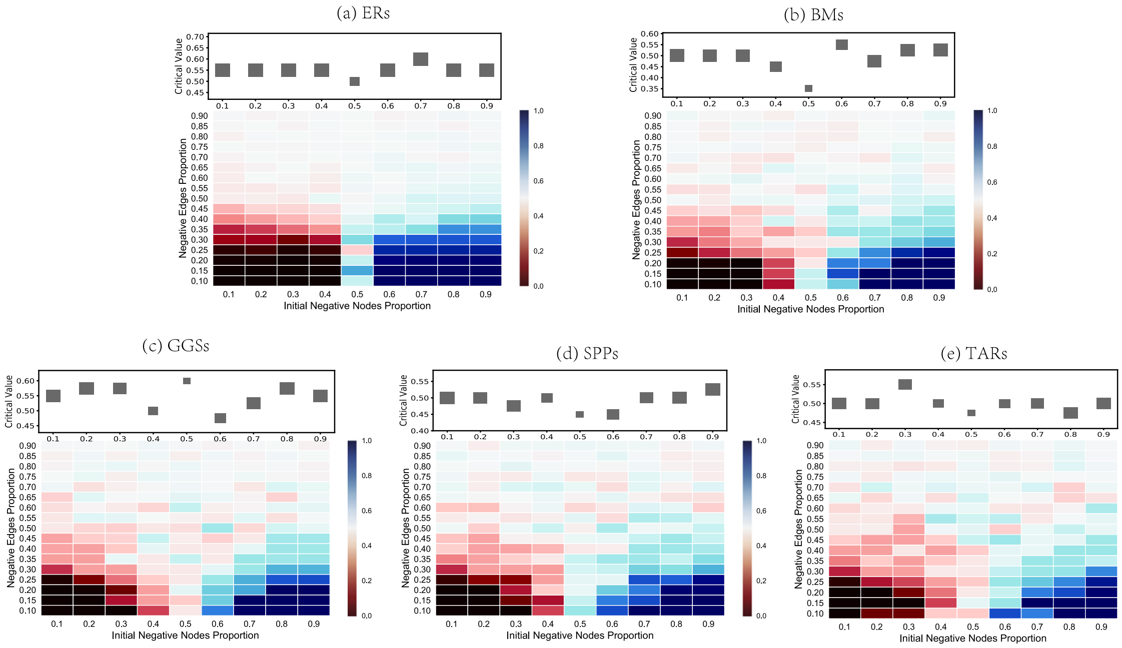

Fig. 4. The evolutionary results on networks with odd numbers of communities. (a)∼(e) represent the results on ER structure, BM structure, GGS structure, SPP structure and TAR structure networks, respectively. The heat maps and bubble maps are generated in the same way as Fig. 3.

And the critical negative edge ratio σ of all five kinds of network structures basically fluctuates around 0.5, but there are differences. In particular, the critical ratios of synthetic signed networks with different community structures are all lower than the critical ratio of synthetic random signed network.

Take Fig. 3(a) as an example, we can find that when the value of negative edges proportion is too large or too small, the evolution results show a completely different phenomenon according to the heat map, which shows the critical negative edge ratio. And comparing five bubble maps in Fig. 3, the critical negative edge ratio of synthetic random signed network, shown in Fig. 3(a) is significantly higher than all the other four network structures, which indicates the transition of the evolutionary results in random structures requires more negative edge ratio.

(2) *(phenomenon One)* When the negative edge ratio of the network exceeds the critical ratio σ, all the networks present consistent evolutionary result, that is, the negative node proportion is stable at 0.5 or fluctuates around 0.5 when the system reaches the steady state, whatever the initial node proportion holding the negative opinion ρ_0 is.

Take Fig. 3(a) as an example, we can find that when the negative edge ratio exceeds σ, the negative node proportion is almost stable at 0.5. But comparing the color of heat maps in Fig. 3(b) and Fig. 4(b), the color in Fig. 4(b) is messier, indicating the result data of odd number communities is more fluctuant.

The practical significance: In this case, it can be seen clearly that the negative edge ratio of the network is high, which symbolizes that the negative relationship is the dominant driver in the system. The system is full of unstable factors, and people may change their attitude at any time because of suspicion, competition, etc. Therefore, it is more difficult to form a stable friendly alliance in such network, which leads to more liberalized opinion competition in the network. The distribution of opinions is more scattered, and the proportions of positive and negative opinions are more equal, and then, a stable situation will be formed that is not affected by the proportion of opinions in the initial state.

(3) *(phenomenon Two)* When the negative edge ratio of the network is below the critical ratio σ, all the networks basically presented a positive correlation between negative node proportion in steady state and the proportion in initial state, for example shown in Fig. 3(a).

The practical significance: In this case, it can be seen clearly that the low proportion of negative edges in the network symbolizes that the positive relationship in the system dominates, and people tend to form larger groups due to the widespread friendly relationship. It is easier to form stable and reliable alliance, which leads to small groups uniting and holding one consistent attitude, so that the opinion with initial high support rate has always maintained its advantages, and eventually formed a kind of evolution where the stronger is bound to be stronger.

(4) The transition of evolution results on synthetic signed networks with community structures is gentler than that on synthetic random signed network. This can be concluded by comparing the color changes in Fig. 4(a) and other subplots in Fig. 4, and also observing bigger bubbles in Fig. 4(a) than other subplots in Fig. 4, and the same in Fig. 3.

What is more, we also observe that even if the evolution results of proportion look the seem, the distributions of nodes states are quite different in signed networks with different structures. The stricter the community structure of the signed network is, the more consistent the states of nodes from the same community tend to be. For example, with all other parameters the same, the proportion of positive and negative nodes in one community tend to be 1:1 in signed networks with BM structure, while in signed networks with TAR structure, all the nodes in one community tends to be the same states.

3.2. Real-world signed networks

We further test our method by applying it to two real-world signed networks. As shown in Fig. 5(a), the first real-world network is the Gahuku–Gama subtribes network, which is based on the study of highland New Guinea culture [22], where nodes are sub-tribes, with positive edges representing the political alliance between sub-tribes, and negative edges representing the hostile relationship between sub-tribes. The negative edge ratio of the network is 0.5 and most of the edges between communities are negative, while only positive edges exist within communities. As shown in Fig. 5(b), the second real-world network is a relation graph of 10 parties of the Slovene Parliamentary in 1994 [23]. The nodes are political parties, and the edges represent the similarities between the two parties. The relationship between the parliamentary parties is estimated by 72 members of the Slovenian national assembly through questionnaires. The original network is a weighted signed network, and in this work, the network is transformed into a corresponding unweighted network for research. The negative edge ratio of this network is 0.6, and most of the edges within communities are positive, while only negative edges exist between communities.

Apply our model to these two signed networks, with the results shown in Fig. 6. The evolutionary results of the Gahuku–Gama subtribes network (shown in Fig. 6(a)) indicates that the proportion of nodes holding the negative opinion at steady state is roughly between 0.4 and 0.6, and positively correlated with the proportion at initial state slightly. This represents that for subtribes network with three communities, the negative edges, which accounts for half of the edges, will tend to balance the system. The support gained by the initially dominant opinion or organization will be dispersed, and the power gap between subtribes will gradually narrow, but the gap will not disappear entirely.

While in the Slovene parliamentary party network (shown in Fig. 6(b)), the proportion of nodes holding the negative opinion at steady state was stable at 0.5, independent of the proportion at initial state. This network has an even number of communities and a larger negative edges proportion, which is different from the subtribal network above. In this case, the competition between the parties is stronger, the interest conflicts are more complicated, and the hostile or competitive relationship is more common. Therefore, the opinions of the parties on a certain issue are more likely to change, resulting in a greater volatility of the system and the final dynamic balance. This is consistent with the checks and balances between the various parties in the real world.

In addition, the simulation on corresponding synthetic random signed networks of the two real-world networks have also been carried out. These random signed networks have the same node number, average degree and negative edge proportion as the real networks. The results show that the steady-state negative node proportion of synthetic random

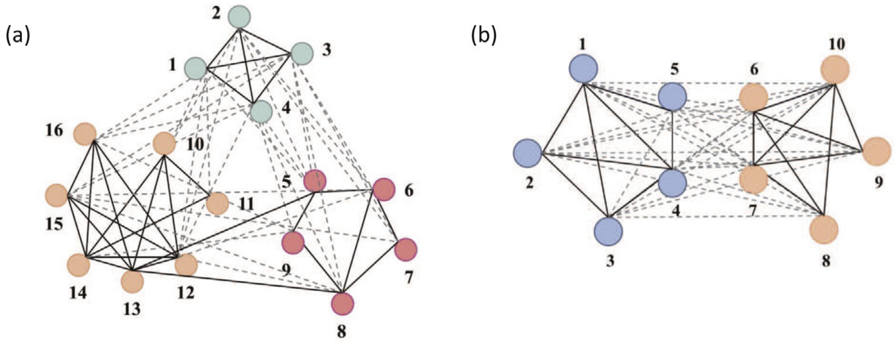

Fig. 5. The diagram of the two real networks. (a) The Gahuku–Gama subtribes network with 16 nodes, whose negative edge ratio is 0.5 and most of the edges between communities are negative, but only positive edges exist within communities. (b) The Slovene Parliamentary party network with 10 nodes, whose negative edge ratio is 0.6, and most of the edges within communities are positive, but only negative edges exist between communities.

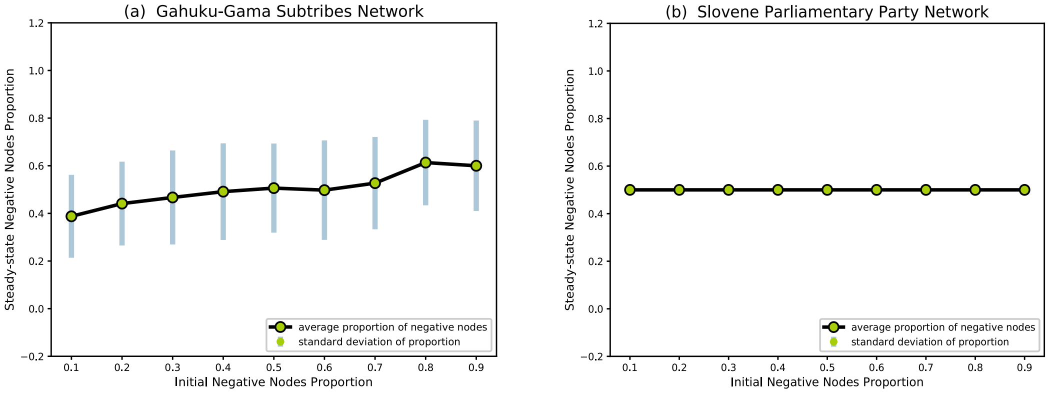

Fig. 6. The evolutionary results on two real networks. (a) The left figure represents the results on the Gahuku–Gama subtribes network, with the length of the blue bar representing the magnitude of standard deviation. (b) The right figure represents the results on the Slovene Parliamentary party network.

signed network fluctuates around 0.5. However, the result of the corresponding random signed network of subtribes network fluctuates more slightly than subtribes network, while the reverse is true for party network with even number of communities.

It can be seen that for the evolution of the networks in the real world, the conclusions obtained by numerical simulation of synthetic networks are equally applicable.

3.3. Network structural balance

In order to further study the global function of negative edges in the evolution of public opinion, experiments about the network balance are also carried out [24]. Structural balance theory, which was put forward in the 1940s by Heider, is an important social theory for signed networks. We can refer to the calculation method proposed in the literature [25] to measure network structural balance. The mechanism used in this work can be seen in Fig. 7.

Table 1 shows the values of some indicators about network structure balance on five synthetic networks and two real networks. According to the calculation results, the following conclusions are obtained:

(1) According to the data of triples with nodes and edges, it can be seen that along with the evolution, the proportion of unbalanced motif decreases, shown in Fig. 8(a). Therefore with public opinion evolving, the networks can achieve a great degree of system stability and balance at the same time. Moreover, the proportion of unbalanced motif in real networks decreases significantly more than that in the synthetic networks.

(2) In real networks, the proportion of unbalanced triples with only edges (shown in Table 1) is smaller than that of synthetic networks, but the proportion of unbalanced triples with edges and nodes is larger. That indicates there are less abnormal relationships existing in real social networks, but the opinion spreading is more complex, where it is more likely to accept friends holding opposing opinions and enemies potentially holding the same views, shown in Fig. 8. As mentioned in [18], the fraction of balanced triangles appear more frequently than unbalanced ones in real-world networks (see Fig. 8).

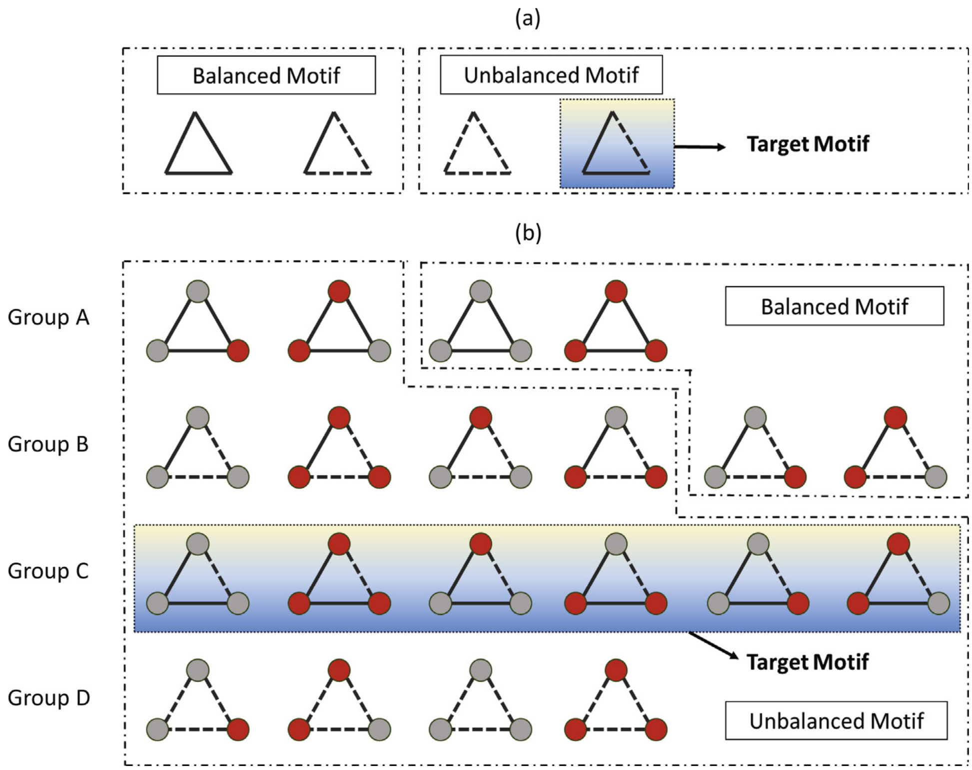

Fig. 7. Balanced and unbalanced configurations of a triple. (a) The top part shows triples that contain only edges, where solid or dotted line represent positive or negative edge. (b) The bottom part shows triples that contain nodes and edges. The red node and gray node represent nodes holding different opinions. These twenty triples are divided into four groups, and group C is the target triple that we care about.

Table 1
The network structure balance data of five kinds of synthetic networks and two real networks.

Network	Triples only with edges			Triples with nodes and edges			
				Initial networks		Steady-state networks	
	Unbalanced motif proportion	Target motif proportion	Target motif/ Unbalanced motif	Unbalanced motif proportion	Target motif/ Unbalanced motif	Unbalanced motif proportion	Target motif/ Unbalanced motif
ER structure	0.502	0.373	0.744	0.875	0.426	0.729	0.512
BM structure	0.411	0.294	0.716	0.790	0.373	0.617	0.477
GGS structure	0.207	0.154	0.747	0.625	0.247	0.370	0.417
SPP structure	0.366	0.171	0.466	0.801	0.213	0.561	0.305
TAR structure	0.094	0.000	0.000	0.616	0.000	0.255	0.000
Real GGS	0.132	0.029	0.222	0.725	0.041	0.272	0.108
Real SPP	0.117	0.033	0.286	0.835	0.040	0.125	0.267

4. Conclusion

In this paper, we discuss the binary opinion dynamics on signed networks based on Ising Model. In complex real-world systems, individuals interact with each other by amicable or antagonistic relationships, and the public opinion evolves towards some direction to make the system more stable. And to model the dynamic process, we apply the Ising model to signed networks as an evolutionary mechanism of binary opinion. The interaction between individuals is represented by the relationship between two nodes in the theoretical model. We observe the influence of the network structure on the evolutionary results of public opinion and focus on the function of the proportion and location of negative edges in signed networks.

In order to adapt the research on signed networks, the Hamiltonian function of classical Ising model is modified. The evolution model is validated both on the synthetic and on the real-world signed networks. It can be seen that there is a critical value of negative edges proportion in signed networks. When the negative edges proportion crosses the critical ratio, the evolution of the public opinion on signed network will appear two distinct results. Specifically, when the negative edges ratio is higher than the critical ratio, the evolutionary result is independent of the initial state of opinion and is

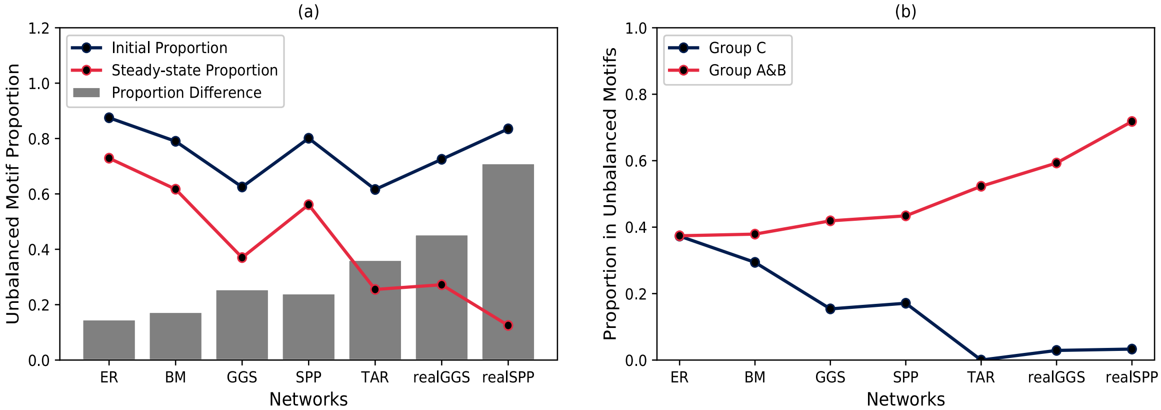

Fig. 8. The comparison of network structural balance. (a) The comparison of unbalanced motif proportion at initial state (blue line) and steady state (red line), with absolute value of the difference represented by gray bars. (b) The comparison of the proportion of unbalanced motif in group A and B (blue line) and group C (red line). All the unbalanced motifs we mentioned here refer to the triples with nodes and edges.

basically stable in a constant state; when the negative edges ratio is lower than the critical ratio, the evolutionary result has a certain degree of positive correlation with the initial state of opinion, and the lower the negative edges ratio is, the stronger the correlation appears. Moreover, although the critical ratio fluctuates around a constant, the location of the negative edges in signed network affects the specific value of the critical ratio.

The network structural balance calculation has also performed. The results show that there are less abnormal relationships existing in real social networks, but the opinion spreading is more complex. But even if there are such differences, when the network public opinion evolves according to the model proposed in this work, both real and synthetic signed networks will constantly improving the network structural balance while pursuing the lowest system energy.

In this work, we apply the classic Ising model to the signed networks. Since the Ising model is a relatively simple opinion evolutionary mechanism, more sophisticated models should be applied to signed networks for further research. For example, the modified Ising models and the Potts model representing polymorphism can be used in the future.

Acknowledgments

This work is supported by the National Natural Science Foundation of China (Grant Nos. 61573065 and 71731002).

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